T* Lite: A Fast Time-Risk Optimal Motion Planning Algorithm for Multi-Speed Autonomous Vehicles

James P. Wilson†, Zongyuan Shen†, Shalabh Gupta†, Thomas A. Wettergren‡

†Laboratory of Intelligent Networks and Knowledge-Perception Systems
Department of Electrical & Computer Engineering, University of Connecticut, Storrs, CT, USA
‡Naval Undersea Warfare Center, Newport, RI 02841, USA

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Autonomous Vehicle Path Planning Applications

**Mine Countermeasures**


**Search & Rescue**

US Coast Guard Office of Search and Rescue (CG-SAR)

**Oil & Gas Industry**

“Saab Sabertooth AUV/ROV for Oil & Gas Inspection”, Saab, Jan 26, 2018

**Wildlife Habitat Monitoring**

Deep-sea coral specimen collection. Plymouth University et al. 2018
Introduction

Objective: Develop a computationally efficient time-risk optimal motion planner for variable-speed autonomous vehicles in obstacle-rich environments.

Existing Approaches

T* for Time-Risk Optimal Motion Planning
- The T* algorithm [1] is the only motion planner that considers multi-speed vehicles and jointly optimizes time and risk.
- Limitation: Grid-based approach is computationally expensive

Sample-based Methods for Rapid Motion Planning
- RRT* and PRM* [2] quickly generate asymptotically-optimal shortest paths as number of samples increase
- Limitation: Restricted to single-speed vehicles & no risk considered

Kinodynamic Motion Models
- Dubins [3] provides shortest paths for single velocity vehicles
  - Limitation: Does not consider multi-speed vehicles
  - Limitation: Requires nonlinear solvers
- The recently developed Generalized Multi-speed Dubins Motion Model (GMDM) [5] overcomes the above limitations:
  - Better maneuvering by controlling the turning radius
  - Speed selected based on obstacle distance to mitigate risk
  - Allows for real-time computation

Features and Contributions of T*-Lite
- Enables fast time-risk optimal motion planning for variable-speed vehicles by:
  - Porting the novel time-risk cost function from T* into a fast and asymptotically-optimal sample-based motion planner
  - Generating samples from a four-dimensional configuration space considering position, heading, and speed.
  - Utilizing the GMDM to produce the optimal time-risk trajectories connecting sampled states
- Algorithm is computationally efficient while providing reasonable solution quality
T*-Lite Overview

(a) Overview of the computation of the time and risk costs in the joint optimization problem.

(b) Example of the high-dimensional sampled vehicle states and the time-risk optimal trajectory produced by the Generalized Multi-speed Dubins Motion Model.
Autonomous Vehicle Description

- \((x, y, \theta) \in SE(2)\) is the vehicle and position heading
- Taking speed \(v(t)\) and turning rate \(u(t)\) be the inputs, the equations of motion are:
  \[
  \begin{align*}
  \dot{x}(t) &= v(t) \cdot \cos \theta(t) \\
  \dot{y}(t) &= v(t) \cdot \sin \theta(t) \\
  \dot{\theta}(t) &= u(t)
  \end{align*}
  \]

**Bounded Turning Rate**

\(u(t) \in [-u_{\text{max}}, u_{\text{max}}] \text{ rad/s}, u_{\text{max}} \in \mathbb{R}^+\) is the max turning rate, and "-/+" indicates a right/left turn

**Variable Speed**

\(v(t) \in [v_{\text{min}}, v_{\text{max}}] \text{ m/s}\)

**Curvature**

\(\kappa(t) = \frac{u(t)}{v(t)}, \quad 0 \leq |\kappa(t)| \leq \frac{u_{\text{max}}}{v_{\text{min}}}\)

Note: curvature is the inverse of turning radius: \(r(t) = \frac{1}{\kappa(t)}\)

Search Area: \(A \in \mathbb{R}^2\)

- Obstacle Space \(A_{\text{obs}}\)
- Free Space \(A_{\text{free}}\)

Define vehicle state as \(p = (x, y, \theta, v)\)

- \((x, y) \in A_{\text{free}}\)
- \(\theta \in [0, 2\pi]\)
- \(v \in [v_{\text{min}}, v_{\text{max}}]\)
Admissible Control: Let $\Gamma$ denote the set of collision-free paths between the start state $p_{\text{start}}$ and goal state $p_{\text{goal}}$. For each path $\gamma \in \Gamma$, the control $c(s) = (\kappa, v)$ at any point $s$ on path $\gamma$, belongs to:

$$\Omega = \{(\kappa, v): v_{\min} \leq v \leq v_{\max}, |\kappa| \leq \frac{u_{\max}}{v}\}$$

Cost of a Path: Let $R(s)$ denote the risk cost at point $s$ on path $\gamma$. Then the total cost is written as:

$$J(\gamma) = \int_{\gamma} R(s) \frac{1}{v(s)} ds$$

Objective: Find the optimal control $c^* \in \Omega$, which generates the collision-free path $\gamma^*$, such that: $J(\gamma^*) \leq J(\gamma)$, $\forall \gamma \in \Gamma$ in a computationally efficient manner.
Approximate Piecewise Path Cost Function: Assume a constant risk along path $\gamma_{i,i+1}$. Thus:

$$ J(\gamma_{i,i+1}) = R(\gamma_{i,i+1}) \cdot \int_{\gamma_{i,i+1}} \frac{1}{v(s)} ds $$

Risk Cost $R(\gamma_{i,i+1})$: For each evenly interpolated state $\hat{p}_\ell$ along $\gamma_{i,i+1}$:

1. Compute collision time $t_\ell = \frac{d_\ell}{v_\ell}$
2. Given safety threshold $t^*$, compute sample risk:

$$ risk(\hat{p}_\ell) = \begin{cases} 
1 + \log\left(\frac{t^*}{t_\ell}\right) & \text{if } t_\ell < t^* \\
1 & \text{if } t_\ell \geq t^*
\end{cases} $$

Finally, the piecewise risk is computed as:

$$ R(\gamma_{i,i+1}) = \max_{\ell \in \{1,\ldots,M\}} (risk(\hat{p}_\ell))^k $$

$k > 0$ is the user-defined risk weight

An example of the interpolated state sequence $P^m$ composed of states $p^m_i = (x_i, y_i, \theta_i, v_i)$
The **Generalized Multi-speed Dubins Motion Model (GMDM)** is a fundamental improvement of the Dubins model that enables the selection of any speed for any of the three segments of a Dubins path ($L \equiv$ left turn, $S \equiv$ straight, or $R \equiv$ right turn).

- Used to generate a set of candidate trajectories that connect any two states $p_i$ to $p_{i+1}$

**Main Features:**

- Provides path planners the flexibility to select the appropriate speed dynamically based on the perceived risk
- Selection of both turning rate and speed enables selection of appropriate turning radius to smoothly maneuver around obstacles based on their shapes and sizes
- Synthesis is similar to Dubins, thus providing simple closed-form solutions for real-time computation.

Visualization of the Generalized Multi-speed Dubins Motion model for each of the six path types.
**T*-Lite Algorithm**

**RRT* Motion Planner**

T*-Lite utilizes the asymptotically-optimal sample-based RRT* framework, which has six core functions:

- Nearest neighbor
- Near-by vertices
- Collision check
- Sampling
- Distance
- Local steering

**Summary of RRT***

- Sampled states are randomly generated in the obstacle-free space
- A search tree of minimum-cost collision-free paths that connect states to the start node is created
- As new states are added, connections between existing states are updated if connections to the new states are faster.

**Figure**: Illustration of RRT* and the iterative search-tree update for a point vehicle. Note: connections between nodes in T*-Lite are subject to curvature constraints.
\( \text{T}^*\text{-Lite Algorithm} \)

\section*{Core Functions}

\( \text{T}^*\text{-Lite} \) is based on the asymptotically-optimal sample-based RRT* framework, which has six core functions:

- Nearest neighbor
- Near-by vertices
- Collision check
- Sampling
- Distance
- Local steering

\textbf{Sampling Function:} generates randomly sampled collision-free states \( \mathbf{p} = (x, y, \theta, v) \in \mathbf{P} \) in the obstacle-free space \( \mathcal{A}_{free} \).

\textbf{Distance Function:} Let \( \text{dist}: \mathbf{P} \times \mathbf{P} \to \mathbb{R}^2 \) be a function that returns the cost of the time-risk optimal trajectory \( \gamma^*_{i,i+1} \) between two states \( \mathbf{p}_i, \mathbf{p}_{i+1} \in \mathbf{P} \) such that \( \text{dist}(\mathbf{p}_i, \mathbf{p}_{i+1}) = J(\gamma^*_{i,i+1}) \).

\textbf{Local Steering Function:} Given two states \( \mathbf{p}_i, \mathbf{p}_{i+1} \in \mathbf{P} \), the \( \text{steer} \) function produces the optimal collision-free trajectory \( \gamma^*_{i,i+1} \) connecting \( \mathbf{p}_i \) to \( \mathbf{p}_{i+1} \) such that \( J(\text{steer}(\mathbf{p}_i, \mathbf{p}_{i+1})) = \text{dist}(\mathbf{p}_i, \mathbf{p}_{i+1}) \). Producing the optimal trajectory requires:

- The \text{approximate optimization function from } \text{T}^* \text{ to evaluate the time-risk costs of the created candidate trajectories}
- A kinodynamic motion model to create a sufficient set of candidate trajectories connecting two states
Simulation Setup

Autonomous Vehicle and $T^*$-Lite Parameters:
- $(v_{\text{min}}, v_{\text{max}}) = (0.5, 1.0) \text{ m/s}$
- $u_{\text{max}} = 0.5 \text{ rad/s}$
- Safety threshold $t^* = 6 \text{ s}$
- Risk weight $k = 2$
- Num. of interpolated states $M = 4$
- Search tree max size: 3000 sampled states
- Num. of nearest neighbors: 100
- Max. connection distance: $3m$
- Scenario Size: $30m \times 30m$

Motion models used in $T^*$-Lite:
- Max-speed Dubins motion model
- Generalized Multi-speed Dubins Motion model

Figure: Scenario used in simulation
Results

Dubins:
- Travel Time: 44.56 s
- Max Risk: 2.13
- Time-Risk Total: 60.47
- CPU Time: 1.34 s

GMDM:
- Travel Time: 39.23 s
- Max Risk: 1.69
- Time-Risk Total: 45.78
- CPU Time: 8.65 s

Single speed reduces CPU requirements but produces longer and riskier paths.

Variable speeds enhance maneuverability and reduce risk to provide quicker and safer paths.

Velocity States:
- 1.0 m/s
- 0.5 m/s

Risk Values:
- Max 2.5
- Min 1.0
Conclusions & Future Work

Conclusions
- Developed T*-Lite for rapid time-risk optimal motion planning for variable-speed autonomous vehicles.
  - Achieved by:
    - Porting the novel time-risk cost function from T* into the RRT* framework
    - Generating high-dimensional samples that considers vehicle position, heading, and speed.
    - Utilizing the Generalized Multi-speed Dubins Motion model to provide near-optimal trajectories in a computationally efficient manner
  - Provides fast, safe, and flexible maneuvers in obstacle-rich environments
  - Suitable for on-demand real-time motion planning

Future Work
- In-depth analysis of the T*-Lite framework in other asymptotically optimal sample-based frameworks.
- Direct comparisons against the grid-based T* in terms of both solution quality and CPU time.
- Develop smart high-dimensional sampling methods for multi-speed vehicles to further enhance solution quality and reduce computation time.
- Extend to multi-agent resilient systems.